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SEMIFULL LINE (BLOCK) SYMMETRIC n-SIGRAPHS

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Abstract

An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ $(S_n = (G, \mu))$, where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ $(\mu : V \to H_n)$ is a function. In this paper we introduced the new notions semifull symmetric n-sigraph and semifull line (block) symmetric n-sigraph of a symmetric n-sigraph and its properties are obtained. Also, we obtained the structural characterizations of these notions. Further, we presented some switching equivalent characterizations.

1. Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory the reader is refer to [1]. We consider only finite, simple graphs free from self-loops.

Key Words : *Symmetric n-sigraphs, Symmetric n-marked graphs, Balance, Switching, Semifull symmetric n-sigraphs, Semifull line (block) symmetric n-sigraphs.*

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Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m, where $m = \lceil \frac{n}{2} \rceil$.

A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ $(S_n = (G, \mu))$, where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ $(\mu : V \to H_n)$ is a function.

In this paper by an *n-tuple/n-sigraph/n-marked graph* we always mean a symmetric n-tuple/symmetric n-sigraph/symmetric n-marked graph.

An n-tuple $(a_1, a_2, ..., a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. In an n-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity n-tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Further, in an n-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the *n-tuple* $\sigma(A)$ is the product of the n-tuples on the edges of A.

In [10], the authors defined two notions of balance in n-sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P.S.K.Reddy [6]

Definition : Let $S_n = (G, \sigma)$ be an n-sigraph. Then,

(i) S_n is *identity balanced* (or *i-balanced*), if product of n-tuples on each cycle of S_n is the identity n-tuple, and

(ii) S_n is *balanced*, if every cycle in S_n contains an even number of non-identity edges.

Note: An i-balanced n-sigraph need not be balanced and conversely.

The following characterization of i-balanced n-sigraphs is obtained in [10].

Theorem 1.1 (**E. Sampathkumar et al. [10]**) : An n-sigraph $S_n = (G, \sigma)$ is i-balanced if, and only if, it is possible to assign n-tuples to its vertices such that the n-tuple of each edge uv is equal to the product of the n-tuples of u and v.

In [10], the authors also have defined switching and cycle isomorphism of an n-sigraph $S_n = (G, \sigma)$ as follows: (See also [5], [7-9], [12-22]).

Let $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$, be two n-sigraphs. Then S_n and S'_n are said to be *isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that if uv is an edge in S_n with label $(a_1, a_2, ..., a_n)$ then $\phi(u)\phi(v)$ is an edge in S'_n with label $(a_1, a_2, ..., a_n)$.

Given an n-marking μ of an n-sigraph $S_n = (G, \sigma)$, *switching* S_n with respect to μ is the operation of changing the n-tuple of every edge uv of S_n by $\mu(u)\sigma(uv)\mu(v)$. The n-sigraph obtained in this way is denoted by $\mathcal{S}_\mu(S_n)$ and is called the *μ-switched n-sigraph* or just *switched n-sigraph*.

Further, an n-sigraph S_n *switches* to n-sigraph S'_n (or that they are *switching equivalent* to each other), written as $S_n \sim S'_n$, whenever there exists an n-marking of S_n such that $\mathcal{S}_\mu(S_n) \cong S'_n$.

Two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ are said to be *cycle isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that the n-tuple $\sigma(C)$ of every cycle C in S_n equals to the n-tuple $\sigma(\phi(C))$ in S'_n.

We make use of the following known result (see [10]).

Theorem 1.1 (**E. Sampathkumar et al. [10]**) : Given a graph G, any two n-sigraphs with G as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.

Let $S_n = (G, \sigma)$ be an n-sigraph. Consider the n-marking μ on vertices of S defined as follows: each vertex $v \in V$, $\mu(v)$ is the product of the n-tuples on the edges incident at v. *Complement* of S is an n-sigraph $\overline{S_n} = (\overline{G}, \sigma')$, where for any edge $e = uv \in \overline{G}$, $\sigma'(uv) = \mu(u)\mu(v)$. Clearly, $\overline{S_n}$ as defined here is an i-balanced n-sigraph due to Theorem 1.1.

If $B = \{u_1, u_2, \cdots, u_r, r \geq 2\}$ is a block of a graph Γ, then we say that vertex u_1 and block B are incident with each other, as are u_2 and B and so on. If two blocks B_1 and B_2 of G are incident with a common cut vertex, then they are adjacent blocks. If $B = \{e_1, e_2, \cdots, e_s, s \geq 1\}$ is a block of a graph G, then we say that an edge e_1 and block B are incident with each other, as are e_2 and B and so on. This concept was introduced by Kulli [2]. The vertices, edges and blocks of a graph are called its members.

The line graph $L(G)$ of a graph G is the graph whose vertex set is the set of edges of G in which two vertices are adjacent if the corresponding edges are adjacent (see [1]).

The semifull graph $\mathcal{SF}(G)$ of a graph G is the graph whose vertex set is the union of vertices, edges and blocks of G in which two vertices are adjacent if the corresponding members of G are adjacent or one corresponds to a vertex and the other to an edge incident with it or one corresponds to a block B of G and the other to a vertex v of G and v is in B. In fact, this notion was introduced by Kulli [3].

In [4], the author introduced the new notions called "Semifull line graphs and semifull block graphs" as follows: The semifull line graph $\mathcal{SFL}(G)$ of a graph G is the graph whose vertex set is the union of the set of vertices, edges and blocks of G in which two vertices are adjacent $\mathcal{SFL}(G)$ if the corresponding vertices and edges of G are adjacent or one corresponds to a vertex of G and other to an edge incident with it or one corresponds to a block B of G and other to a vertex v of G and v is in B.

The semifull block graph $\mathcal{SFB}(G)$ of a graph G is the graph whose vertex set is the union of the set of vertices, edges and blocks of G in which two vertices are adjacent in $\mathcal{SFB}(G)$ if the corresponding vertices and blocks of G are adjacent or one corresponds to a vertex of G and other to an edge incident with it or one corresponds to a block B of G and other to a vertex v of G and v is in B.

2. Semifull Line n-Sigraph of an n-Sigraph

Motivated by the existing definition of complement of an n-sigraph, we now extend the notion called semifull line graphs to realm of n-sigraphs: the *semifull line n-sigraph* $\mathcal{SFL}(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ as an n-sigraph $\mathcal{SFL}(S_n) = (\mathcal{SFL}(G), \sigma')$, where for any edge e_1e_2 in $\mathcal{SFL}(G)$, $\sigma'(e_1e_2) = \sigma(e_1)\sigma(e_2)$. Further, an n-sigraph $S_n = (G, \sigma)$ is called semifull line n-sigraph, if $S_n \cong \mathcal{SFL}(S'_n)$ for some n-sigraph S'_n. The following result indicates the limitations of the notion of semifull line n-sigraphs as introduced above, since the entire class of i-unbalanced n-sigraphs is forbidden to be semifull line n-sigraphs.

Theorem 2.1 : For any n-sigraph $S_n = (G, \sigma)$, its semifull line n-sigraph $\mathcal{SFL}(S_n)$ is i-balanced.

Proof : Since the n-tuple of any edge uv in $\mathcal{SFL}(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n, by Theorem 1.1, $\mathcal{SFL}(S_n)$ is i-balanced. $\square$

For any positive integer k, the k^{th} iterated semifull line n-sigraph, $\mathcal{SFL}^k(S_n)$ of S_n is defined as follows:

$$\mathcal{SFL}^0(S_n) = S_n,\ \mathcal{SFL}^k(S_n) = \mathcal{SFL}(\mathcal{SFL}^{k-1}(S_n)).$$

Corollary 2.2 : For any n-sigraph $S_n = (G, \sigma)$ and for any positive integer k, $\mathcal{SFL}^k(S_n)$ is i-balanced.

Theorem 2.3 : For any two n-sigraphs S_n and S'_n with the same underlying graph, their semifull line n-sigraphs are switching equivalent.

Proof : Suppose $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ be two n-sigraphs with $G \cong G'$. By Theorem 2.1, $\mathcal{SFL}(S_n)$ and $\mathcal{SFL}(S'_n)$ are i-balanced and hence, the result follows from Theorem 1.2. □

The *semifull n-sigraph* $\mathcal{SF}(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ as an n-sigraph $\mathcal{SF}(S_n) = (\mathcal{SF}(G), \sigma')$, where for any edge e_1e_2 in $\mathcal{SF}(G)$, $\sigma'(e_1e_2) = \sigma(e_1)\sigma(e_2)$. Further, an n-sigraph $S_n = (G, \sigma)$ is called a semifull n-sigraph, if $S_n \cong \mathcal{SF}(S'_n)$ for some n-sigraph S'_n. The following result indicates the limitations of the notion of semifull n-sigraphs as introduced above, since the entire class of i-unbalanced n-sigraphs is forbidden to be semifull n-sigraphs.

Theorem 2.4 : For any n-sigraph $S_n = (G, \sigma)$, its semifull n-sigraph $\mathcal{SF}(S_n)$ is i-balanced.

Proof : Since the n-tuple of any edge uv in $\mathcal{SF}(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n, by Theorem 1.1, $\mathcal{SF}(S_n)$ is i-balanced. □

For any positive integer k, the k^{th} iterated semifull n-sigraph, $\mathcal{SF}^k(S_n)$ of S_n is defined as follows:

$$\mathcal{SF}^0(S_n) = S_n,\ \mathcal{SF}^k(S_n) = \mathcal{SF}(\mathcal{SF}^{k-1}(S_n)).$$

Corollary 2.5 : For any n-sigraph $S_n = (G, \sigma)$ and for any positive integer k, $\mathcal{SF}^k(S_n)$ is i-balanced.

Theorem 2.6 : For any two n-sigraphs S_n and S'_n with the same underlying graph, their semifull n-sigraphs are switching equivalent.

Proof : Suppose $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ be two n-sigraphs with $G \cong G'$. By Theorem 2.4, $\mathcal{SF}(S_n)$ and $\mathcal{SF}(S'_n)$ are i-balanced and hence, the result follows from Theorem 1.2. □

In [4], the author characterizes graphs such that semifull line graphs and semifull graphs are isomorphic.

Theorem 2.7 : Let $G = (V, E)$ be a nontrivial connected graph. The graphs $\mathcal{SFL}(G)$ and $\mathcal{SF}(G)$ are isomorphic if and only if G is a block.

In view of the above result, we have the following result that characterizes the family of n-sigraphs satisfies $\mathcal{SFL}(S_n) \sim \mathcal{SF}(S_n)$.

Theorem 2.8 : For any n-sigraph $S_n = (G, \sigma)$, $\mathcal{SFL}(S_n) \sim \mathcal{SF}(S_n)$ if and only if G is a block.

Proof : Suppose that $\mathcal{SFL}(S_n) \sim \mathcal{SF}(S_n)$. Then clearly, $\mathcal{SFL}(G) \cong \mathcal{SF}(G)$. Hence by Theorem 2.7, G is a block.

Conversely, suppose that S_n is an n-sigraph whose underlying graph is a block. Then by Theorem 2.7, $\mathcal{SFL}(G)$ and $\mathcal{SF}(G)$ are isomorphic. Since for any n-sigraph S_n, both $\mathcal{SFL}(S_n)$ and $\mathcal{SF}(S_n)$ are i-balanced, the result follows by Theorem 1.2. □

The following result characterize signed graphs which are semifull line n-sigraphs.

Theorem 2.9 : An n-sigraph $S_n = (G, \sigma)$ is a semifull line n-sigraph if and only if S_n is i-balanced n-sigraph and its underlying graph G is a semifull line graph.

Proof : Suppose that S_n is an i-balanced and G is a semifull line graph. Then there exists a graph G' such that $\mathcal{SFL}(G') \cong G$. Since S_n is i-balanced, by Theorem 1.1, there exists an n-marking ζ of G such that each edge uv in S_n satisfies $\sigma(uv) = \zeta(u)\zeta(v)$. Now consider the n-sigraph $S'_n = (G', \sigma')$, where for any edge e in G', $\sigma'(e)$ is the n-marking of the corresponding vertex in G. Then clearly, $\mathcal{SFL}(S'_n) \cong S_n$. Hence S_n is a semifull line n-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a semifull line n-sigraph. Then there exists an n-sigraph $S'_n = (G', \sigma')$ such that $\mathcal{SFL}(S'_n) \cong S_n$. Hence, G is the semiful line graph of G' and by Theorem 2.1, S_n is i-balanced. □

In view of the above result, we can easily characterize n-sigraphs which are semifull n-sigraphs.

3. Semifull Block n-Sigraph of an n-Sigraph

Motivated by the existing definition of complement of an n-sigraph, we now extend the notion called semifull block graphs to realm of n-sigraphs: the *semifull block n-sigraph* $\mathcal{SFB}(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ as an n-sigraph $\mathcal{SFB}(S_n) = (\mathcal{SFB}(G), \sigma')$, where for any edge e_1e_2 in $\mathcal{SFB}(G)$, $\sigma'(e_1e_2) = \sigma(e_1)\sigma(e_2)$. Further, an n-sigraph $S_n = (G, \sigma)$ is called a semifull block n-sigraph, if $S_n \cong \mathcal{SFL}(S'_n)$ for some signed graph S'_n. The following result indicates the limitations of the notion of semifull block n-sigraphs as introduced above, since the entire class of i-unbalanced n-sigraphs is forbidden to be semifull block n-sigraphs.

Theorem 3.1 : For any n-sigraph $S_n = (G, \sigma)$, its semifull block n-sigraph $\mathcal{SFB}(S_n)$ is i-balanced.

Proof : Since the n-tuple of any edge uv in $\mathcal{SFB}(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n, by Theorem 1.1, $\mathcal{SFB}(S_n)$ is i-balanced. $\square$

For any positive integer k, the k^{th} iterated semifull block n-sigraph, $\mathcal{SFB}^k(S_n)$ of S_n is defined as follows:

$$\mathcal{SFB}^0(S_n) = S_n,\ \mathcal{SFB}^k(S_n) = \mathcal{SFB}(\mathcal{SFB}^{k-1}(S_n)).$$

Corllary 3.2 : For any n-sigraph $S_n = (G, \sigma)$ and for any positive integer k, $\mathcal{SFB}^k(S_n)$ is i-balanced.

Theorem 3.3 : For any two n-sigraphs S_n and S'_n with the same underlying graph, their semifull block n-sigraphs are switching equivalent.

Proof : Suppose $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ be two n-sigraphs with $G \cong G'$. By Theorem 3.1, $\mathcal{SFB}(S_n)$ and $\mathcal{SFB}(S'_n)$ are i-balanced and hence, the result follows from Theorem 1.2. $\square$

In [4], the author characterizes graphs such that semifull block graphs and semifull graphs are isomorphic.

Theorem 3.4 : Let $G = (V, E)$ be a nontrivial connected graph. The graphs $\mathcal{SFB}(G)$ and $\mathcal{SF}(G)$ are isomorphic if and only if G is P_2.

In view of the above result, we have the following result that characterizes the family of n-sigraphs satisfies $\mathcal{SFB}(S_n) \sim \mathcal{SF}(S_n)$.

Theorem 3.5 : For any n-sigraph $S_n = (G, \sigma)$, $\mathcal{SFB}(S_n) \sim \mathcal{SF}(S_n)$ if and only if G is P_2.

Proof : Suppose that $\mathcal{SFB}(S_n) \sim \mathcal{SF}(S_n)$. Then clearly, $\mathcal{SFB}(G) \cong \mathcal{SF}(G)$. Hence by Theorem 3.4, G is P_2.

Conversely, suppose that S_n is an n-sigraph whose underlying graph is P_2. Then by Theorem 3.4, $\mathcal{SFB}(G)$ and $\mathcal{SF}(G)$ are isomorphic. Since for any n-sigraph S_n, both $\mathcal{SFB}(S_n)$ and $\mathcal{SF}(S_n)$ are i-balanced, the result follows by Theorem 1.2. $\square$

In view of the Theorem 2.9, we can easily characterize signed graphs which are semifull block signed graphs.

4. Conclusion

We have introduced the new notions for n-signed graphs called semifull n-sigraph and semifull line (block) n-sigraph of an n-sigraph. We have proved some results and presented the structural characterization of these notions. There are no structural characterizations of semifull graph and semifull line (block) graph, but we have obtained the structural characterizations of semifull n-sigraph and semifull line (block) n-sigraph.

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